FLEXIBLE LIFE ANNUITIES

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Abstract

Annuity contracts typically deliver a stream of income at a predetermined level in order to insure against the risk of longevity. This paper explores whether flexible annuities, which give subscribers the possibility to choose between different levels for their annuities, may be welfare enhancing. In the case where agents gradually discover their actual probability of survival, a predetermined and "one-size-fits-all" annuity plan is optimal. If an expenditure risk is added along with the longevity risk, a flexible annuity plan is better even though the consumption path cannot be isolated from uninsured expenses anymore.

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It is a well established fact that very few assets are converted into life annuities outside of social security and traditional defined benefit pension plans. In the United States for example, 401(k) pension plans are now the dominant form of private pension. But very few 401(k) plans offer the option to annuitize. More than 50% of households do not expect to even partly convert their defined contribution account balances into an annuity (Brown, 2001). The United Kingdom has a long history of mandatory annuitization during retirement. In particular, defined contribution pension funds are currently required to provide 75-year-old retirees with 75 percent of pension assets in the form of annuities. However, less and less insurers are willing to offer such products. In France, a voluntary and fully funded personal savings account called PERP has been recently launched where buyers are compelled to fully annuitize the wealth accumulated in the plan at the date of retirement. Up to now, this constraint seems to hinder its commercial extension.

The underinvestment in life annuities contrasts with the benefits such annuities are expected to deliver to retirees. Yaari (1965) shows that full annuitization of assets is optimal in a standard model of saving without a bequest motive. Davidoff, Brown, and Diamond (2005) show an identical result for a large set of preferences and environments. A central question is therefore why the annuity market is so small. A preliminary answer is that even though life annuities provide unequalled insurance against longevity, they also have particular disadvantages. Most obviously, the purchaser loses control over his assets, as most annuity plans deliver a predetermined lifelong income. Hence, once annuitized, wealth cannot serve to absorb unexpected income shocks, where most of such shocks for the retirees are health-related. The risk of facing lower consumption following a period of unexpected health expenditures is recognized as being detrimental to the demand of annuities (Brown, 2004). It has also a significant impact on saving behavior. For instance, several articles argue that the existence of out-of-pocket medical expense risk explains why the elderly run down their assets so slowly (Palumbo 1999, De Nardi et al. 2006)

At first sight, this drawback could be minimized by providing the annuitant with some flexibility over the annuity profile. A flexible contract can be defined as a contract that gives the annuitant the possibility to withdraw a higher annuity in case of an expenses shock. The absence of such an option in existing contracts is commonly justified by an argument of adverse selection as stressed by Brown and Warshawsky (2001, p.14): "Insurance companies do not allow individuals to cancel an annuity agreement once it is in place. Otherwise adverse selection would obviously occur as individuals acquire information about their expected longevity." If individuals update their actual chance of longevity, annuitants who expect to live longer would indeed like to increase their financial stake in the plan while the shorter-lived consumers would like to consume greater amounts earlier, thereby reducing their annuitized wealth. If the insurer gives the subscribers the flexibility to choose between different levels at which they consume their annuity, the expected return of a pool of saving can be greatly reduced by an adverse shift in the composition of the population of subscribers. At the extreme limit, if agents were always informed sufficiently in advance of the day they die, they would have the time to close their personal account, thereby escaping the redistribution scheme. This would cause the collapse of the longevity insurance mechanism.

The objective of this paper is to explore this issue in depth and to ask whether the adverse selection argument in itself prevents any flexibility in annuity plans from being feasible. I begin by presenting a simple model where life expectancy is the only source of uncertainty. Agents update their mortality risk during the retirement phase. This raises the question whether they should modify the level of their annuity according to their more precise estimate of mortality. It is shown that in this simple model, they should not. A fixed annuity plan which provides a predetermined stream of annuities independent of future information about longevity is optimal because it prevents annuitants to draw their wealth down in case of bad news about their mortality. This is a clear case where flexible life annuities are not welfare enhancing in line with the argument stressed by Brown and Warshawsky (2001).

Next, the previous model is extended by including a liquidity risk in addition to the longevity risk. Some savers face a liquidity need due to adverse shocks to their health. Some flexibility through a choice between different levels of annuities then becomes optimal. However, the risk of increased expenses cannot be fully insured, due to the adverse selection mechanism just outlined.

The main result of the analysis is that adverse selection *per se* does not justify the complete absence of flexibility. The choice of raising an annuity during one period up to a limit may cover some additional expenses, and thereby provide insurance. Flexibility is then optimal, even though a reduction in the rate of return arises due to adverse selection. This result is important, given the general lack of annuitization observed in most developed countries. Indeed, a minimal degree of flexibility may reduce the mismatch between the desired consumption path and the annuity income stream, thereby promoting wealth annuitization.

The literature on the annuity market has developed rapidly for two decades.

Most of the studies focus on the "annuity puzzle" by examining why households do not buy more annuity contracts when they retire. The studies generally assume an exogenous annuity stream and examine which type of consumers' preferences or which market imperfections allow to explain why the theoretical prediction of full annuitization (Yaari, 1965) does not seem to hold empirically (e.g. Kotlikoff and Spivak 1981, Walliser 1999, Milevsky and Young 2002, Davidoff, Brown, and Diamond 2005). Those articles differ in perspective from the present work by taking the annuity stream offered by the insurer as given. In contrast, the annuity plan is endogenous here in the sense that its optimal design depends on the presence of various types of uncertainty.

The issue addressed by the present article is formally close to Brugiavini (1993). He studies at which stage of their life individuals should purchase annuities given that they learn about their survival probability as they get older. His model predicts a purchase in an early stage of the life cycle. The present model displays similar implications if the period during which individuals update their survival probability is reinterpreted as a retirement period instead of an early working period. However, a different question is addressed in the two papers. His paper asks whether workers should buy an annuity contract at an early stage of their life and if they do, whether they could recontract when they retire. She finds that workers could not purchase more annuities later in life since it would reveal a high risk for insurers. In the present model, the early purchase of a fixed annuity works as a commitment device that prevents savers to consume more in case of bad news about their longevity. A formal connection between the two setups is explored further at the end of Section 2.

In section 1 a simple model of annuity demand when longevity is uncertain is introduced. The basic argument of why a fixed annuity plan may be optimal is provided. Section 2 incorporates a liquidity shock and studies to what extent this additional source of uncertainty may be optimally insured by a flexible annuity plan. The last section concludes.

1 A model with uncertainty about longevity

In this section, a formal case against flexible annuities is presented in a basic setting with a longevity risk only. Subscribers are not allowed to choose their annuity profile on a period by period basis. This case will serve as a benchmark model when a second source of uncertainty will be introduced in the next section.

Consider an environment where agents allocate their wealth w between consumption at two dates $t = 1, 2$. The gross interest rate of the economy is denoted by R. Life expectancy is uncertain so that individuals face a probability to die before their last period consumption. The uncertainty of survival calls for the purchase of annuities which pay at a premium when alive in exchange for the subscribers' wealth upon death. In the absence of a bequest motive or uninsurable risk, full annuitization prevails (Yaari, 1965). The role of the insurer is to pool individuals with similar longevity expectations but varying longevity outcomes as a means to protect them against the longevity risk.

In the model, uncertainty about the mortality risk gradually resolves itself between a preliminary date 0 and date 1. Individuals have the same life expectancy at $t = 0$. At $t = 1$, they obtain more precise information about their actual probability to survive the last period $t = 2$. A fraction p of annuitants learn that their probability to survive is π^h. The remaining annuitants are characterized by a lower survival probability $\pi^l < \pi^h$. c_{ti} denotes consumption at date $t = 1, 2$ of agents with the longevity type $i = h, l$. Utility of consumption per period is denoted $u(c)$, with $u'(c) > 0$, $u''(c) < 0$ and $\lim u'(c) = \infty$ when $c \to 0$. Intertemporal utility is additive and β is the subjective discount rate attached to the last period utility:

$$v(c_{1i}, c_{2i}; \pi_i) = u(c_{1i}) + \pi_i \beta u(c_{2i})$$

Information about longevity is private. Hence, insurers are unable to separate the annuitants by risk classes. Insurance firms cannot monitor whether costumers hold annuities also from other firms. They compete by offering the most attractive rate of return for saving. Individuals can purchase as many annuities as they want at the prevailing rate of return. This leads to the definition of an asymmetric information equilibrium (e.g. Abel, 1986) characterized by the annuity plan $\{(\widehat{c}_{1i}, \widehat{c}_{2i}); i = h, l\}$ which satisfies:

$$\begin{aligned} (i) \qquad (\widehat{c}_{1i}, \widehat{c}_{2i}) &= \arg \left\{ \begin{array}{c} \max v(c_{1i}, c_{2i}; \pi_i) \\ \text{s.t. } c_{1i} + \overline{\pi} c_{2i}/R = w \end{array} \right\} \qquad i = h, l \\ (ii) \qquad \overline{\pi} &= \frac{p\widehat{c}_{2h}}{p\widehat{c}_{2h} + (1-p)\widehat{c}_{2l}} \pi_h + \frac{(1-p)\widehat{c}_{2l}}{p\widehat{c}_{2h} + (1-p)\widehat{c}_{2l}} \pi_l \end{aligned} \tag{1}$$

Insurers provide annuitants with the highest actuarially feasible rate of return $R/\overline{\pi}$. This rate takes as given the average mortality risk $\overline{\pi}$, weighted by the participation rate of each type in the annuity plan. Given this rate of return, consumers choose the best annuity profile according to the above definition.

Now, assume that the consumers have the possibility to sign a binding contract at date 0 at a time when they have not yet updated their information about their longevity. They leave their saving to the insurers, which maximizes their date 0 expected utility stream:

$$\left\{ \begin{array}{c} \max pv(c_{1h}, c_{2h}; \pi_h) + (1-p)v(c_{1l}, c_{2l}; \pi_l) \\ \text{s.t. } p(w - c_{1h} - \pi_h c_{2h}/R) + (1-p)(w - c_{1l} - \pi_l c_{2l}/R) = 0 \end{array} \right. \tag{2}$$

The resulting consumption path $\{(c^*_{1i}, c^*_{2i}); i = h, l\}$ is defined by:

$$\begin{cases} c^*_{th} = c^*_{tl} \qquad t = 1, 2 \\ u'(c_{1i}) = \beta R u'(c_{2i}) \qquad i = 1, 2 \end{cases} \tag{3}$$

The allocation is Pareto optimal since information is symmetric at date 0. It equalizes consumption across risks and makes the consumption path independent of the mortality risk.

In this equilibrium, insurers compete for savings as soon as date 0 by promising an annuity sequence $\{(c^*_{1i}, c^*_{2i}); i = h, l\}$. It remains to see whether individuals are willing to immediately buy such a contract or prefer to postpone their consumption decision. Indeed, even though this contract is optimal from an ex ante perspective, this is obviously not a contract that retirees would buy at date 1 once their individual longevity is better assessed. Proposition 1 states that the date 0 expected utility associated with buying the ex ante optimal contract is higher than the expected utility derived from postponing the consumption decision (proof in Appendix):

Proposition 1. $pv(c^*_{1h}, c^*_{2h}; \pi_h) + (1-p)v(c^*_{1l}, c^*_{2l}; \pi_l) > pv(\widehat{c}_{1h}, \widehat{c}_{2h}; \pi_h) + (1-p)v(\widehat{c}_{1l}, \widehat{c}_{2l}; \pi_l)$.

There is a basic intuition behind this result, first outlined by Hirshleifer (1971). The reduction of uncertainty at date 1 prevents any transfer from the low risk agents to the high risk agents, which undermines the insurance scheme.

Proposition 1 is a clear case against flexible life annuities since it establishes the superior efficiency of an unconditional annuity plan that delivers a single level of annuity at dates 1 and 2. By not providing agents with the possibility to receive a higher annuity in case of a bad signal about their longevity, a fixed saving plan preserves a higher rate of return, and allows to mutualize the risk of longevity more efficiently.

Even though it is optimal to buy a predetermined annuity plan at date 0 according to proposition 1, it remains to check that agents cannot improve their situation by recontracting at date 1 by buying or selling new annuities. Let us denote by b_i the additional amount of annuities that they buy (or sell short) at time 1 for the last period consumption. Because they have invested all their saving at date 0 in the predetermined annuity plan, they use the payment c_1^* to consume and save at date 1: $\widetilde{c}_{1i} + b_i = c_1^*$ so that their last date consumption if alive is $\widetilde{c}_{2i} = c_2^* + Rb_i/\pi_i$. The rate of return is adjusted to each specific risk since buying new annuities reveals a low risk and short selling reveals a high risk[1]. Hence, the revised consumption plan is determined by:

$$\left\{ \begin{array}{c} \max v(c_{1i}, c_{2i}; \pi_i) \\ \text{s.t. } c_{1i} + \pi_i c_{2i}/R = c_1^* + \pi_i c_2^*/R \end{array} \right\} \qquad i = h, l$$

and does not deviate from the ex ante optimal allocation, which means that $b_i = 0$. This sums up the argument put forth by Brugiavini (1993). Agents do not recontract later since the effect of new information about longevity on consumption is exactly offset by the revision of the actuarially fair interest rate.

This section has shown the optimality of a fixed annuity plan. The next section adds uninsurable expenses, which leads to a different result.

[1] Short selling could be ruled out without changing the revelation mechanism outlined.

2 Adding uninsurable expense shocks

The previous framework is preserved except that some uninsurable expenses, possibly related to health, may be incurred at date 1. There are now three sorts of consumers who discover their type at date 1. The first one, represented in proportion p, is a long-lived consumer who survives date 2 with probability $\pi > 0$. Such agents are denoted as of type h below. Agents of a second type m are represented with a proportion q and also survive with probability π, but incur an additional cost $m > 0$ at date 1. This cost is interpreted as an out-of-pocket medical expenditure. Last, type l agents, in proportion $s = 1 - p - q$, are short-lived consumers who die with certainty at the end of period 1.

Insurers offer an annuity plan at date 0, at a time when agents do not know their type and some insurance is still possible. An optimal plan is a set of annuities $\{(c_{1h}, c_{2h});\ (c_{1m}, c_{2m});\ (c_{1l}, 0)\}$ dedicated to types h, m and l respectively.

It is informative to start the analysis with the hypothesis of complete information, where the three types are publicly known at date 1. Annuity contracts can be tailored to each type and full insurance between the insured prevails: $c^*_{1h} = c^*_{1m} - m = c^*_{1l}$ and $c^*_{2h} = c^*_{2m}$. The risk of longevity and the expense shock are perfectly mutualized across agents. Moreover, consumption evolves in accordance with the first best allocation: $u'(c^*_{1i}) = \beta R u'(c^*_{2i})$, $i = h, l$.

In the more realistic case in which an agent's type is private information, the insurer has to make sure that each annuity plan is chosen by the type for whom it is designed. A first constraint checks that type h agents actually choose the annuity plan (c_{1h}, c_{2h}) instead of (c_{1m}, c_{2m}) designed for type m individuals. This happens if the first type does not achieve a higher utility by selecting the annuity c_{1m}, saving the amount $x = c_{1m} - c_{1h}$ and then consuming $c_{2h} = c_{2m} + xR/\pi$ in the last period. The possibility to switch to a different annuity profile leads to a reservation level of utility denoted by v_h:

$$v_h(z) = \{\max u(c_1) + \pi\beta u(c_2); c_1 + c_2\pi/R = z\} \tag{4}$$

where $z = c_{1m} + \pi c_{2m}/R$ is the intertemporal value of the annuity plan (c_{1m}, c_{2m}). The incentive compatibility constraint is then:

$$u(c_{1h}) + \pi\beta u(c_{2h}) \geq v_h(z) \tag{5}$$

Note that a first best policy with full information consists of a redistribution of resources to the agents who incur the expense shock because $c_{1h} = c_{1m} - m < c_{1m}$ and $c_{2h} = c_{2m}$. When the type is private information, the insurer is constrained to give the same actuarial value to both types (see the resource constraint in (4)). For the same reason, the reverse constraint in which a type m would prefer (c_{1h}, c_{2h}) to (c_{1m}, c_{2m}) is not operative in equilibrium as the insurance scheme implies transferring more resources to type m, not less.

A second incentive compatibility constraint recognizes that type l agents always choose the highest annuity available at date 1. It follows that the annuity c_{1l} cannot be less than the same period annuity designed for long-lived agents: $c_{1l} \geq \max(c_{1h}, c_{1m})$.

An equilibrium set of contracts maximizes the consumers' expected utility given a global resource constraint and the two information constraints:

$$\left\{ \begin{array}{ll} \max p(u(c_{1h}) + \pi\beta u(c_{2h})) + q(u(c_{1m} - m) + \pi\beta u(c_{2m})) + su(c_{1l}) & \\ & p(c_{1h} + \pi c_{2h}/R) + q(c_{1m} + \pi c_{2m}/R) + sc_{1l} = w \\ \text{s.t.} & u(c_{1h}) + \pi\beta u(c_{2h}) \geq v_h(z) \\ & c_{1l} \geq \max(c_{1h}, c_{1m}) \end{array} \right. \tag{6}$$

It is straightforward to show that $c_{1m} \geq c_{1h}$ in equilibrium. If this were not the case, the insurer could marginally reduce the date 1 consumption of type 1 and increase the consumption of type 2 by a factor p/q yielding a net utility increase of $u'(c_{1m}-m)-u'(c_{1h}) > 0$. Hence, it follows that the last information constraint simplifies to

$$c_{1l} \geq c_{1m} \tag{7}$$

The program (6) can be rewritten in a more convenient way:

$$\begin{cases} \max p(u(c_{1h}(z)) + \pi\beta u(c_{2h}(z))) + q(u(c_{1m} - m) + \pi\beta u(c_{2m})) + su(c_{1l}) \\ \text{s.t.} \quad (1-s)(c_{1m} + \pi c_{2m}/R) + sc_{1l} = w \\ \qquad c_{1l} \geq c_{1m} \end{cases}$$

where $c_{1h}(z)$ and $c_{2h}(z)$ are agents' optimal demands of annuities solving (4). The second constraint merges with the budget constraint of (4) into the broader budget constraint of the insurer in (6). We can next examine the consequences for the optimal annuity plan.

Proposition 2. The incentive compatibility constraint (7) is binding: $c_{1l} = c_{1m}$.

The basic reason behind this inequality is that an expense shock is a legitimate reason to let the saver withdraw a higher annuity whereas a bad signal about survival is not. As a result, the insurer would like to offer a smaller annuity to a type l compared to the one proposed to a type m. As distinguishing between the two types is impossible, the insurer is compelled to offer a single annuity level for both types.

A natural question that arises is whether the annuitants suffering an expense shock benefit at all from an insurance mechanism, given the adverse selection mechanism discussed in the introduction. The following proposition gives a positive answer:

Proposition 3. $c_{1m} > c_{1h}$.

This proposition shows that individuals can rely on a higher annuity when they must meet additional expenditures. This option comes at the cost of a diminished annuity in the last period since the information constraint (5) binds. Proposition 3 addresses to what extent this increases the insurance mechanism.

Proposition 4. The annuity plan (c_{1m}, c_{2m}) satisfies $u'(c_{1m} - m) > R\beta u'(c_{2m})$.

This proposition shows that annuitants who experience an expense shock cannot withdraw a sufficiently high annuity in order to fully smooth their consumption profile compared to the first best environment. Indeed, the insurer is prevented from raising the annuity because it magnifies the adverse selection effect and undermines the longevity insurance mechanism as previously discussed. On the contrary, perfect consumption smoothing could be restored if the adverse selection effect were absent:

Proposition 5. If $s = 0$, $u'(c_{1m} - m) = R\beta u'(c_{2m})$.

To sum up, the insurer proposes three annuity plans $\{(c_{1h}, c_{2h})$; (c_{1m}, c_{2m});

$(c_{1l}, 0)\}$, one for each type. A certain degree of flexibility is optimal, as agents are free to choose between consuming a high annuity c_{1m} and a low one c_{1h}. Choosing the high annuity reduces the last period income but improves the well-being of the agents who face an expense shock at date 1. Short-lived agents choose to "close" their annuity plan by choosing the high level $c_{1l} > c_{1h}$ as well. However, they face a high penalty as they cannot consume more than the level of the annuity that agents with an expense shock are allowed to withdraw. This penalty is optimal as it limits the scope of the adverse selection effect. Since insurers do not allow short-lived agents to deplete their account at date 1, agents in need of additional income cannot fully smooth their consumption profile.

3 Conclusion

This paper studies the optimal annuity contract when agents face a liquidity risk and a longevity risk. Simultaneously analyzing the effects of both types of risk is relevant. Taken separately, they produce conflicting predictions about the characteristics of the contract. With both types of risk, flexible annuities which provide annuitants with the possibility to choose between different withdrawal levels are welfare enhancing as they allow agents suffering the expense shock to better smooth their consumption path.

It is worth noting that flexibility does not mean total freedom for the annuitants. Indeed, the high pay-out level cannot be raised too much, as it leads the short-lived individuals to consume too much, thereby undermining the longevity insurance scheme. This constraint prevents agents suffering the expense shock from perfectly smoothing their consumption profile.

The main result of the paper is that the adverse selection effect sometimes cited against flexible annuities is not sufficient for the optimality of a fixed and "one size fits all" annuity scheme. The option to increase withdrawals in one period up to a limited level may achieve a good deal of consumption smoothing while limiting the size of the rate of return reduction due to the adverse selection effect. A practical conclusion is that a minimal degree of flexibility could well promote wealth annuitization by reducing the mismatch between the desired consumption path and the annuity income stream.

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Appendix

Proof of Proposition 1. By way of comparison, the date 0 problem in (2) can be recast into:

$$\left\{ \begin{array}{c} \max pv(c_{1h}, c_{2h}; \pi_h) + (1-p)v(c_{1l}, c_{2l}; \pi_l) \\ \text{s.t. } p(w - c_{1h} - \overline{\pi}c_{2h}/R) + (1-p)(w - c_{1l} - \overline{\pi}c_{2l}/R) = 0 \\ \overline{\pi} = \frac{pc_{2h}}{pc_{2h}+(1-p)c_{2l}}\pi_h + \frac{(1-p)c_{2l}}{pc_{2h}+(1-p)c_{2l}}\pi_l \end{array} \right.$$

This program closely resembles the ex post problem with asymmetric information defined in (1):

$$\begin{array}{llcl} (i) & \{(\widehat{c}_{1i}, \widehat{c}_{2i}); i = h, l\} & = & \arg \left\{ \begin{array}{c} \max pv(c_{1h}, c_{2h}; \pi_h) + (1-p)v(c_{1l}, c_{2l}; \pi_l) \\ \text{s.t. } c_{1i} + \overline{\pi}c_{2i}/R = w \qquad i = h, l \end{array} \right\} \\ & (ii) \qquad\qquad \overline{\pi} & = & \frac{p\widehat{c}_{2h}}{p\widehat{c}_{2h} + (1-p)\widehat{c}_{2l}}\pi_h + \frac{(1-p)\widehat{c}_{2l}}{p\widehat{c}_{2h} + (1-p)\widehat{c}_{2l}}\pi_l \end{array}$$

except that the budget constraint is now split into two separate constraints instead of one and that the impact of the consumption choice on the rate of return $R/\overline{\pi}$ is not internalized. Hence, despite the objective being the same, the ex post problem includes additional constraints. As a result, it delivers a less efficient consumption stream from the the perspective of date 0.

Proof of Proposition 2.

If $c_{1l} \neq c_{1m}$, the program of the insurer is:

$$\begin{cases} \max p(u(c_{1h}(z)) + \pi\beta u(c_{2h}(z))) + q(u(c_{1m} - m) + \pi\beta u(c_{2m})) + su(c_{1l}) \\ \text{s.t.} \quad (p+q)(c_{1m} + \pi c_{2m}/R) + sc_{1l} = w \end{cases}$$

Forming the Lagrangian function:

$$\begin{aligned} L(c_{1m}, c_{2m}, c_{1l}) &= p(u(c_{1h}(z)) + \pi\beta u(c_{2h}(z))) + q(u(c_{1m} - m) + \pi\beta u(c_{2m})) + su(c_{1l}) \\ &\quad + \lambda[w - (p+q)(c_{1m} + \pi c_{2m}/R) - sc_{1l}] \end{aligned}$$

and setting the partial derivatives to zero with respect to c_{1m} and c_{1l}:

$$\begin{aligned} \frac{\partial L}{\partial c_{1m}} &= p[u'(c_{1h})\frac{\partial c_{1h}}{\partial z} + \pi\beta u'(c_{2h})\frac{\partial c_{2h}}{\partial z}] + qu'(c_{1m} - m) - (p+q)\lambda = 0 \\ \frac{\partial L}{\partial c_{1l}} &= su'(c_{1l}) - s\lambda = 0 \end{aligned}$$

Then, the following inequalities can be derived:

$$\begin{aligned} u'(c_{1l}) &= \frac{p}{p+q}[u'(c_{1h})\frac{\partial c_{1h}}{\partial z} + \pi\beta u'(c_{2h})\frac{\partial c_{2h}}{\partial z}] + \frac{q}{p+q}u'(c_{1m} - m) \\ &= \frac{p}{p+q}u'(c_{1h})[\frac{\partial c_{1h}}{\partial z} + \frac{\pi}{R}\frac{\partial c_{2h}}{\partial z}] + \frac{q}{p+q}u'(c_{1m} - m) \qquad \text{(i)} \\ &= \frac{p}{p+q}u'(c_{1h}) + \frac{q}{p+q}u'(c_{1m} - m) \qquad \text{(ii)} \\ &\geq \frac{p}{p+q}u'(c_{1m}) + \frac{q}{p+q}u'(c_{1m}) = u'(c_{1m}) \qquad \text{(iii)} \end{aligned}$$

Equality (i) is obtained by using the Euler equation $u'(c_{1h}) = \beta Ru'(c_{2h})$ derived from (4), (ii) by differentiating the budget constraint (4): $c_{1h}(z) +$

$c_{2h}(z)\pi/R = z$ and substituting for $\partial c_{1h}/\partial z$. Inequality (iii) exploits the fact that $c_{1m} \geq c_{1h}$ at equilibrium. The result is that $c_{1l} \leq c_{1m}$ or $c_{1l} = c_{1m}$ when the information constraint (7) is taken into account.

Proof of Proposition 3.

Let us prove that $c_{1m} = c_{1h}$ cannot hold at the equilibrium. This implies $c_{2m} = c_{2h}$ from (4). The insurer's program with $c_{1h} = c_{1m} = c_{1l}$ is then:

$$\begin{cases} \max(1-q)u(c_{1m}) + qu(c_{1m}-m) + (1-s)\pi\beta u(c_{2m}) \\ \text{s.t. } c_{1m} + (1-s)c_{2m}\pi/R = w \end{cases}$$

Forming the Lagrangian function and setting the partial derivatives to zero leads to the Euler equation: $(1-q)u'(c_{1m}) + qu'(c_{1m}-m) = R\beta u'(c_{2m})$. Hence $u'(c_{1m}) < R\beta u'(c_{2m})$. But $c_{1m} = c_{1h}$ also implies $c_{2m} = c_{2h}$ and $u'(c_{1m}) = R\beta u'(c_{2m})$ from (4), which contradicts the former equation.

Proof of Proposition 4.

The insurer's program with $c_{1l} = c_{1m}$ is:

$$\begin{cases} \max p(u(c_{1h}(z)) + \pi\beta u(c_{2h}(z))) + q(u(c_{1m}-m) + \pi\beta u(c_{2m})) + su(c_{1m}) \\ \text{s.t. } c_{1m} + (1-s)c_{2m}\pi/R = w \end{cases}$$

Forming the Lagrangian function and setting the partial derivatives to zero:

$$\begin{aligned} \frac{\partial L}{\partial c_{1m}} &= pu'(c_{1h})\frac{\partial c_{1h}}{\partial z} + p\pi\beta u'(c_{2h})\frac{\partial c_{2h}}{\partial z} + qu'(c_{1m}-m) + su'(c_{1m}) - \lambda = 0 \\ \frac{\partial L}{\partial c_{2m}} &= pu'(c_{1h})\frac{\partial c_{1h}}{\partial z}\frac{\pi}{R} + p\pi\beta u'(c_{2h})\frac{\partial c_{2h}}{\partial z}\frac{\pi}{R} + q\pi\beta u'(c_{2m}) - (1-s)\frac{\pi}{R}\lambda = 0 \end{aligned}$$

Substituting for the multiplier λ:

$$\begin{aligned} & \frac{p}{1-s}u'(c_{1h})\frac{\partial c_{1h}}{\partial z} + \frac{p}{1-s}\pi\beta u'(c_{2h})\frac{\partial c_{2h}}{\partial z} + \frac{q}{1-s}\beta R u'(c_{2m}) \\ = \; & pu'(c_{1h})\frac{\partial c_{1h}}{\partial z} + p\pi\beta u'(c_{2h})\frac{\partial c_{2h}}{\partial z} + qu'(c_{1m}-m) + su'(c_{1m}) \end{aligned}$$

Using $u'(c_{1h}) = R\beta u'(c_{2h})$:

$$\begin{aligned}\frac{q}{1-s}R\beta u'(c_{2m}) &= qu'(c_{1m}-m) + su'(c_{1m}) \\ &+ u'(c_{1h})\left[p\frac{\partial c_{1h}}{\partial z} + p\frac{\pi}{R}\frac{\partial c_{2h}}{\partial z} - \frac{p}{1-s}\frac{\partial c_{1h}}{\partial z} - \frac{p}{1-s}\frac{\pi}{R}\frac{\partial c_{2h}}{\partial z}\right]\end{aligned}$$

Differentiating $c_{1h}(z) + c_{2h}(z)\pi/R = z$ and substituting for $\partial c_{1h}/\partial z$ leads to the constrained Euler equation:

$$\begin{aligned}R\beta u'(c_{2m}) &= (1-s)u'(c_{1m}-m) + \frac{s(1-s)}{q}u'(c_{1m}) - \frac{ps}{q}u'(c_{1h}) && (8)\\ &< (1-s)u'(c_{1m}-m) + \frac{s(1-s)}{q}u'(c_{1h}) - \frac{ps}{q}u'(c_{1h}) && \text{(i)}\\ &= (1-s)u'(c_{1m}-m) + su'(c_{1h}) \\ &\leq (1-s)u'(c_{1m}-m) + su'(c_{1m}-m) && \text{(ii)}\\ &= u'(c_{1m}-m)\end{aligned}$$

Inequality (i) uses the fact that $c_{1m} > c_{1h}$ and (ii) $c_{1m} - m \leq c_{1h}$. Indeed, $c_{1m} - m > c_{1h}$ would lead to over-insurance. In this case, $u'(c_{1m}-m) < u'(c_{1h}) = R\beta u'(c_{2h})$. Moreover, $c_{1m} > c_{1h}$ implies $c_{2m} < c_{2h}$ from (4) and therefore $R\beta u'(c_{2m}) > R\beta u'(c_{2h})$ implying $u'(c_{1m}-m) < R\beta u'(c_{2m})$. That is, type m agents consume too much at date 1 despite additional expenses. It follows that insurers could improve the position of type m by reducing c_{1m} and raising c_{2m} without violating the type m resource constraint $c_{1m} + \pi c_{2m}/R = w$ and incentive compatibility constraints.

Proof of Proposition 5.

Take the constrained Euler **equation** (8) found in the previous proof and set $s = 0$ and $p + q = 1$.

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